Dryden Lectureship in Research

New Concepts in Control Theory, 1959-1984

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Introduction

CONTROL theory is a branch of applied mathematics that deals with the analysis and synthesis of logic for the control of man-made systems. It is applied to a wide range of fields from aerospace to robotics and economics.

Its applications to aerospace can be divided into four areas:

1) Flight Planning: The determination of a nominal flight path and associated control histories for a given vehicle to accomplish specified objectives with specified constraints.

2) Navigation: The determination of a strategy for estimating the position of a vehicle along the flight path given outputs from specified sensors.

3) Guidance: The determination of a strategy for following the nominal flight path in the presence of off-nominal conditions, wind disturbances, and navigation uncertainties.

4) Control: The determination of a strategy for maintaining angular orientation of the vehicle during the flight that is consistent with the guidance strategy and the vehicle, crew and passenger constraints.

These four categories often overlap. For example, aircraft velocity and angular orientation are coupled, so that guidance and control of aircraft must be considered together.

Dynamic Models

A prerequisite for developing a satisfactory control history or strategy is a satisfactory dynamic model of the system to be controlled (often called the "plant" by control engineers).

Dynamic modeling of aircraft began with Bryan's work in the early 1900's (see Refs. 1 and 2 for excellent descriptions of the history). When computers became reliable and economical in the 1950's, interest in dynamic modeling for simulation and control design increased considerably. References 2, 3, and 4 cover the state of the art in dynamic modeling of aircraft.

Dynamic modeling of spacecraft began in the 1960's and now includes elastic modeling of spacecraft with many connected components. References 4 through 9 cover much of the state of the art in dynamic modeling of spacecraft, boosters, and missiles.

Dynamic models can be inferred from tests run on the plant. A pilot flying a new aircraft develops a "model" of the aircraft in his brain based on his perceptions of its responses ("outputs") to his motions of the controls ("inputs"). In the last ten to twenty years, investigators have learned how to emulate the pilot by constructing dynamic models of an aircraft from recorded input and output histories (see "Identification" to follow). These models are then stored in a computer for later use in synthesizing autopilot logic.

Flight Planning

Flight planning before 1959 consisted of approximating aircraft flight paths by steady-flight arcs, such as climbout, cruise, constant radius turns, and descent. This was called "performance" analysis.[9] Steady arcs are satisfactory approximations for propeller aircraft, but not always for high-performance jet aircraft. For example, the flight path for a jet fighter to climb to a given altitude and speed in minimum time is not approximated well by steady arcs. To predict optimal flight paths without steady-arc approximations requires the use of the calculus of variations (COV), which, in turn, requires the use of digital computers and the development of algorithms for numerical solutions of COV problems. Such algorithms were developed in the early 1960's. Figure 1 shows an optimal climb path for an early version of the F4 aircraft.[10] These algorithms have been refined and improved to the point where they are used routinely now. Reference 11 is an excellent example of the state-of-the-art in algorithms as of 1981.

Figure 2 shows the flight path of a DC-10 to travel 220 n.mi. using minimum fuel.[12] The Boeing 767 has a Flight Management Computer which calculates the flight path for minimum "direct operating cost," and flies the airplane along this path through the autopilot. The algorithms used are based on the work of Erzberger.[12]

An excellent discussion of optimal flight planning (and, more generally, of optimization methods applied to aircraft design problems) to 1981 is given in Ref. 13.

The Concept of Feedback

The word "strategy" appears in the other three of the four categories of aerospace control theory mentioned above. A strategy involves "feedback" of the output to the input, i.e.,

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knowledge of the output is used to change the input to make the output as close as possible to the desired output. The difference between programmed control and feedback control is shown in Fig. 3.

The concept of feedback began nearly two thousand years ago with the water clock,[14] where water depth was kept constant by a float valve. The constant depth kept the flow rate out of the tank constant. This outflow, when going into a vertical glass cylinder, caused the depth of water in the cylinder to increase linearly with time; hence it became a crude clock. The float valve may be interpreted as a mechanical "feedback" of water depth to valve opening, i.e., as water depth increases or decreases from the desired level, the valve opening is decreased or increased. Figure 4 shows a temperature controller built in 1620 that used feedback control.[14] A small increase in temperature in the incubator caused the alcohol to expand which caused the float to rise and close the damper valve a little bit, thus reduced the heat input. Mayr indicates that this ingenious scheme controlled the temperature within 1°F.

Feedback Control to 1959

The synthesis of feedback control logic was a cut-and-try procedure until about fifty years ago. What was needed was a theory that would predict how a proposed feedback scheme would work before it was built. Such a theory is based on an understanding of stability of dynamic systems. This understanding began about 100 years ago when Routh published his famous paper on the stability of linear dynamic systems.[15] Little theory appears to have been used in developing control systems until the late 1930's. Control theory up to 1947 was summarized in a famous book edited by James, Nichols, and Philips.[16] It was written in connection with research carried on during World War II for building automatically tracking radars.

About 1948, an analytical technique called "root locus" was introduced by Evans.[17] It enables one to quickly sketch the locus of roots of a polynomial equation in the complex plane vs a coefficient in the equation. This technique is used to determine feedback gain parameters for satisfactory behavior of controlled systems, and the polynomial equation in this case is the closed-loop characteristic equation of the controlled system.

In the 1950's analog computers became available commercially, making it much easier and cheaper to do the computations required in control analysis and synthesis.

Control theory up to 1959 was based on frequency response, root locus analysis, and a small set of feedback schemes. This "classical" control theory is summarized in the outstanding treatise written by McRuer, Ashkenas, and Graham.[2]

The simplest feedback scheme is "proportional feedback," where the control is moved in proportion to a measurement of the output to be controlled. This was the scheme used by the Sperry Brothers in 1914 when they demonstrated the first aircraft autopilot. In that case, the aircraft was adjusted to straight and level flight by the pilot and then the autopilot was turned on. The deviations of the (elevator, aileron) angles were made proportional to the deviations of the (pitch, roll) angles of the airplane, which were measured by gyros. As an example of proportional feedback, we consider a simple "wings-level" autopilot for a small airplane (the Navion)[18] shown in Fig. 5. If the ailerons are deflected in proportion to the roll angle (sensed by the vertical gyro), the spiral mode of the aircraft is stabilized as shown in Fig. 6a, which is a root locus vs the feedback gain; the dutch roll and roll modes are changed only slightly for the gain shown.

Another simple scheme is "derivative feedback" where the derivative fo the controlled quantity is fedback to the control. As an example, consider a simple "dutch roll damper" autopilot for the Navion. If the rudder is deflected in proportion to the yaw rate (sensed by a rate gyro), the dutch roll mode is stabilized as shown in Fig. 6b, which shows a root locus vs the feedback gain, with the aileron loop closed. The

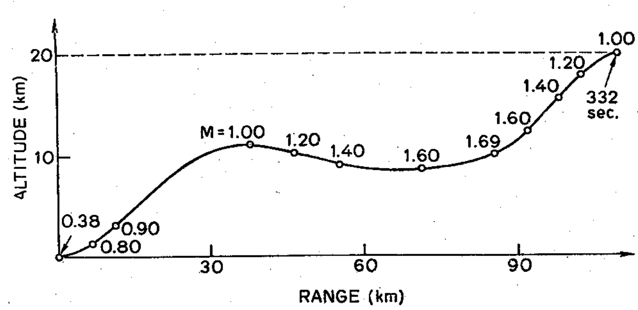


Fig. 1 Flight profile of minimum time to climb path for a supersonic aircraft.

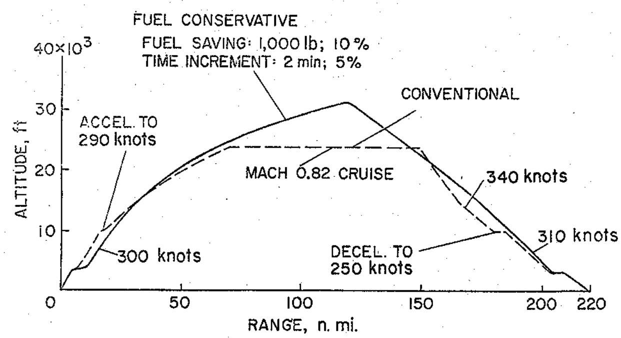


Fig. 2 Flight profile of minimum fuel path for a wide body subsonic transport.[12]

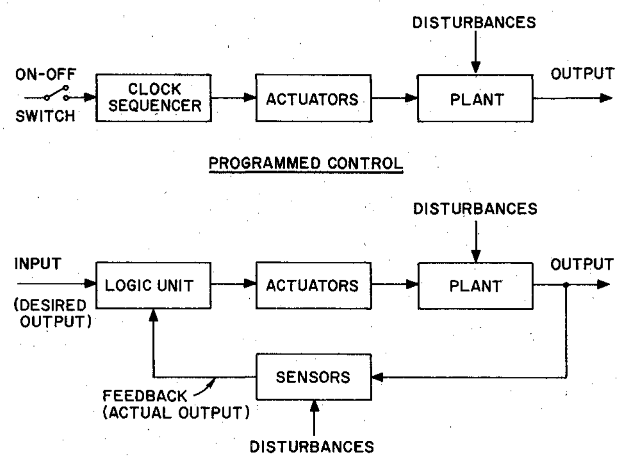


Fig. 3 Block diagrams of programmed and feedback control.

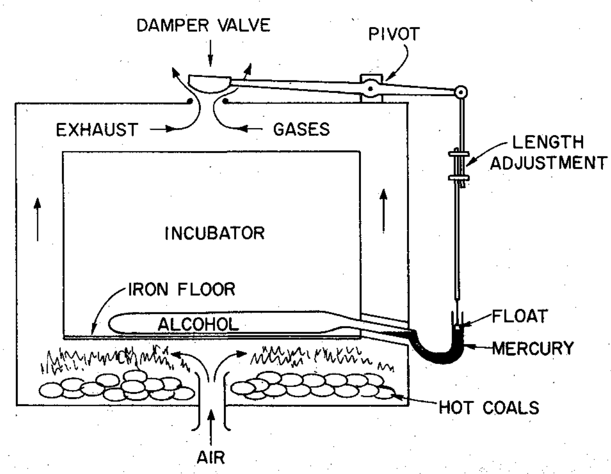


Fig. 4 Temperature regulator of Cornelis Drebbel (1620).[14]

spiral mode is stabilized further while the roll mode is changed only slightly for the gain shown.

Still another simple feedback scheme is "integral feedback" where the integral of the control quantity is fedback to the control. This type of control is particularly useful when three are unknown constant disturbances to the plant. For example, in controlling the rotational speed of a machine, the integral of the difference between the measured speed and the desired speed is fedback to the torque turning the machine.

Combinations of these simple schemes, called PID (proportional, integral, and derivative) control, will handle many simple control problems. To avoid adding a derivative sensor, lead compensation is used to approximate proportional plus derivative feedback. To avoid building up large values on an integrator (which gives initial condition problems), lag compensation is used to approximate proportional plus integral feedback.

State Variable Representations

To simulate a dynamic system on a computer (analog or digital), one needs to arrange the dynamic equations as a set of first order ordinary differential equations since an integration is required for each differentiated variable or "state." If the system is of the nth order, there will be n state variables which form the "state vector" of the system. Using the state vector concept allows one to treat dynamic systems of any order with a simple uniform notation:

$$\dot{x} = f(x,u) \tag{1}$$

$$y = m(x,u) \tag{2}$$

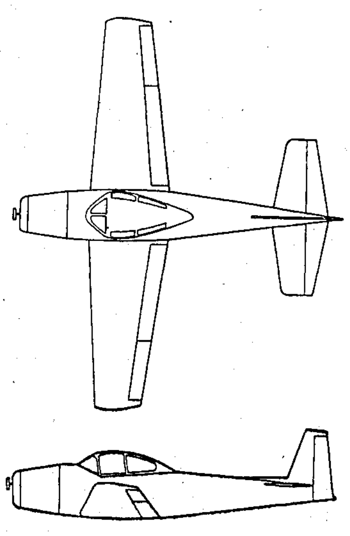
where x is the state vector, u is the control vector (a vector consisting of all the controls of the system), and y is the output vector (a vector consisting of all the outputs of the system).

The model is often linearized about an equilibrium condition. Equations (1) and (2), with $\dot{x} = 0$ and y specified, determine the equilibrium state and control vectors. The linearized dynamic model may then be written as:

$$\dot{x} = F \cdot x + G \cdot u \tag{3}$$

$$y = M \cdot x + L \cdot u \tag{4}$$

where (x,u,y) now represent the DEVIATIONS from the equilibrium (state, control, output) vectors. If there are n states, m controls, and p outputs, then F, G, M, L are coefficient matrices of dimension n-by-n, n-by-m, p-by-n, and p-by-m, respectively. They are partial derivatives of $f(x,u)$ and $m(x,u)$ in Eqs. (1) and (2), evaluated at the reference equilibrium conditions.



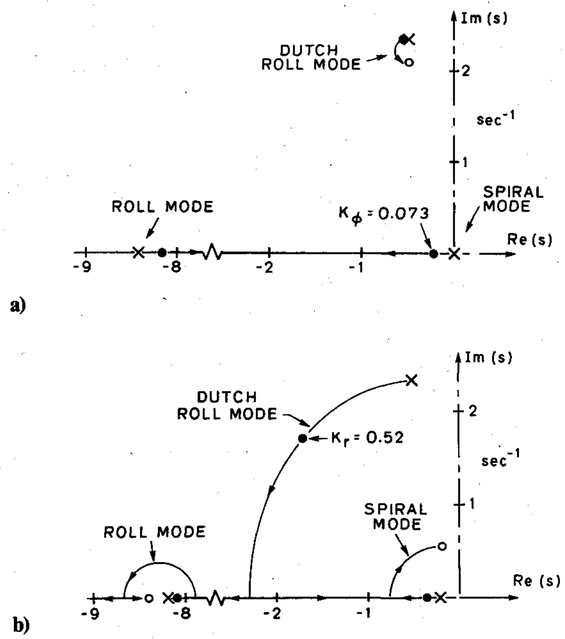


Fig. 6 Navion aircraft: a) locus of closed-loop poles vs gain for feedback of roll angle to a aileron; b) locus of closed-loop poles vs gain for feedback of yaw rate to rudder with aileron loop closed.

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -.089 & -2.19 & .328 & .319 \\ .076 & -.217 & -.166 & 0 \\ -.602 & .327 & -.975 & 0 \\ 0 & .150 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & .0327 \\ .0264 & -.151 \\ .227 & .0636 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \end{bmatrix} + \begin{bmatrix} .089 \\ -.076 \\ .602 \\ 0 \end{bmatrix} \begin{bmatrix} vw \end{bmatrix}$$

$$\begin{bmatrix} \dot{\psi} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2.21 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ y \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix}$$

v = SIDESLIP VELOCITY
r = YAW RATE
p = ROLL RATE
Ø = ROLL ANGLE
ψ = YAW ANGLE
y = LATERAL DISPLACEMENT

δa = AILERON ANGLE
δr = RUDDER ANGLE

vw = LATERAL WIND VELOCITY

UNITS: FT, SEC, .01 RAD

Fig. 7 State variable model of 747 lateral motions in the landing configuration, SAS off.[19]

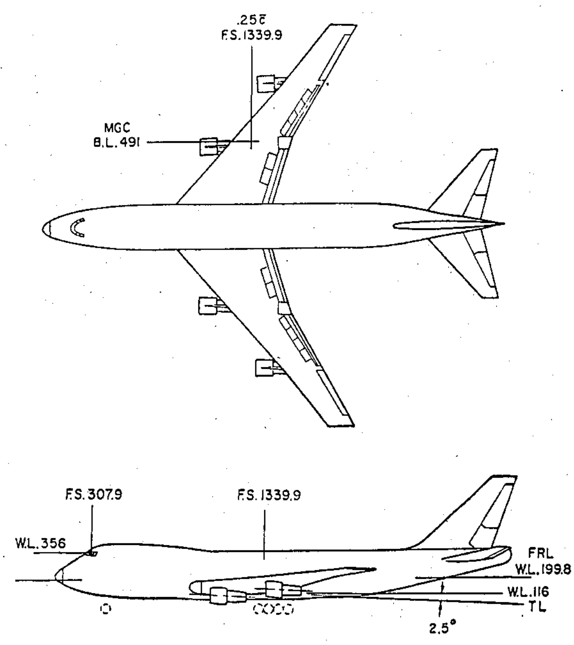


Fig. 8 Top and side views of the 747 aircraft.[18]

Figure 7 shows a linearized state variable model for the lateral motions of the 747 airplane in its landing configuration.[19] Figure 8 shows top and side views of the 747.

Modal Form and Transfer Functions

Having a linearized dynamic model, the control engineer next finds transfer functions from the controls and disturbance inputs to the controlled and measured outputs. A transfer function is the Laplace transform of an output divided by the Laplace transform of an input. Taking the Laplace transform of (3) and (4), we have:

$$y(s) = [L + M\cdot(sI - F)^{-1}\cdot G]\cdot u(s) \tag{5}$$

The coefficient of $u(s)$ in Eq. (5) is a matrix of transfer functions, all having the same denominator polynomial $D(s)$, but different numerator polynomials $N(s)$.

$$\frac{y_i(s)}{u_j(s)} = \frac{Nij(s)}{D(s)} \tag{6}$$

The roots of $Nij(s) = 0$ are called the "zeros" of the transfer function, and the roots of $D(s) = 0$ are called the "poles." Pole-zero diagrams in the complex s-plane are used by the control engineer to decide what type of feedback control logic to use for stabilization.

Another useful form for transfer functions is the partial fraction expansion of Eq. (6):

$$y_i(s) = \sum_j \sum_k \frac{Rij(k)\cdot u_j(s)}{(s - p_k)} \tag{7}$$

where p_k is the pole and $Rij(k)$ is the "residue matrix" at the kth pole for the ith output and the jth input.

The "modal form" of the state representation (3) and (4) is almost exactly in the form (7). The modal form of the lateral motions of the 747 aircraft is shown in Fig. 9. There are five modes:

1) The roll mode with eigenvalues $s = -1.109$ rad/s.
2) The dutch roll mode with complex eigenvalues $s = -0.65 \pm -.731j$ rad/s.
3) The spiral mode with eigenvalue $s = -.043$ rad/s.
4) The heading mode with eigenvalue $s = 0$.
5) The lateral displacement mode with eigenvalue $s = 0$.

Mode 5 is coupled to mode 6 but not vice-versa.

The transfer functions from $(\delta_a, \delta_r, v_w)$ to the state variables (in partial fraction form) can be obtained directly from the modal form, and the residue matrices for each mode are outer products of columns from the output matrix with rows from the input matrix.

State Feedback and Optimal Control

For linear dynamic models, Kalman[320] showed that the "optimal" control vector is a certain linear function of the state vector, i.e., all of the state variables should be fedback to all of the control variables with appropriate gains:

$$u = -C\cdot x \tag{8}$$

where C is the m-by-n feedback gain matrix. By "optimal," he meant that the integral of a quadratic form in y and u is minimized:

$$J = \int_0^\infty (y'Ay + u'Bu)\,dt \tag{9}$$

This method of control synthesis is now called "linear-quadratic" (LQ) synthesis.

$$\begin{bmatrix} m1 \\ m2 \\ m3 \\ m4 \end{bmatrix} = \begin{bmatrix} -1.109 & 0 & 0 & 0 \\ 0 & -.065 & .731 & 0 \\ 0 & -.731 & -.065 & 0 \\ 0 & 0 & 0 & -.043 \end{bmatrix} \begin{bmatrix} m1 \\ m2 \\ m3 \\ m4 \end{bmatrix} + \begin{bmatrix} .174 & -.042 \\ -.076 & .104 \\ .099 & .462 \\ .134 & -.345 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \end{bmatrix} + \begin{bmatrix} .399 \\ -.034 \\ .670 \\ -.004 \end{bmatrix} \begin{bmatrix} vw \end{bmatrix}$$

$$\begin{bmatrix} m5 \\ m6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2.21 & 0 \end{bmatrix} \begin{bmatrix} m5 \\ m6 \end{bmatrix} + \begin{bmatrix} .416 & -1.079 \\ -21.10 & 54.9 \end{bmatrix} \begin{bmatrix} \delta a \\ \delta r \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} vw \end{bmatrix}$$

$$\begin{bmatrix} v \\ r \\ p \\ \phi \\ \psi \\ y \end{bmatrix} = \begin{bmatrix} .309 & 1 & 0 & .169 & 0 & 0 \\ .160 & -.020 & -.209 & .133 & 0 & 0 \\ 1 & -.443 & .281 & -.062 & 0 & 0 \\ -.923 & .392 & .575 & 1 & 0 & 0 \\ -.146 & -.284 & .053 & -3.16 & 1 & 0 \\ .011 & .115 & -.520 & 160 & 0 & 1 \end{bmatrix} \begin{bmatrix} m1 \\ m2 \\ m3 \\ m4 \\ m5 \\ m6 \end{bmatrix}$$

Fig. 9 Modal model of 747 lateral motions in the landing configuration with SAS off.

- PERFORMANCE INDEX $= \int_0^\infty (Ay\cdot y^2 + Aa\cdot ay^2 + \delta a^2 + \delta r^2)\,dt$

  WHERE $ay = -.089v + .0327\delta r$ = LATERAL SPECIFIC FORCE
- USE 1 UNIT (.01 RAD) OF δa & δr IF y = 1 FT OR ay = .1 FT/SEC²

  $Ay = 1$ AND $Aa = 1/(.1)^2 = 100$
- LINEAR-QUADRATIC SYNTHESIS CODE GIVES:

$$\begin{bmatrix} \delta a \\ \delta r \end{bmatrix} = \begin{bmatrix} -1.947 & -3.59 & -1.421 & -1.672 & -7.29 & -.859 \\ 1.263 & 6.42 & .799 & 1.424 & 6.08 & .487 \end{bmatrix} \begin{bmatrix} v \\ r \\ p \\ \phi \\ \psi \\ y \end{bmatrix}$$

Fig. 10 State feedback logic for track hold with coordinated turns; 747 in landing configuration.

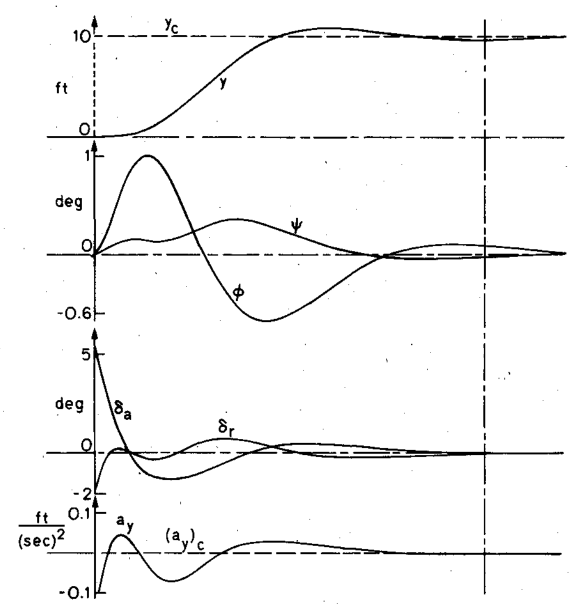


Fig. 11 Response of 747 in landing configuration to command for 10 ft of lateral displacement (sidestep or S-turn maneuver).

If we choose the output and its derivative to be the states for a second order system with one control, then state-feedback control is simply proportional plus derivative control.

Figure 10 shows the optimal feedback gain matrix for the 747 for "track-hold" with coordinated turns. (The real 747 control system was not designed using LQ synthesis.) The choice of the weighting factors Ay and Aa is made on the basis that 1 unit (.01 rad) of aileron and rudder should be used for 1 ft of lateral displacement y, or .1 ft/s² of lateral acceleration.

Figure 11 shows the response of the 747 to a command for 10 ft of lateral displacement (a sidestep or S-turn maneuver). Note the graceful coordinated use of rudder and aileron to produce the maneuver.

Nearly all of the six states are sensed or estimated on-board the 747; v, ϕ, ψ are available from the inertial navigation system (INS); p and r could be estimated by lead networks operating on the ϕ and ψ signals from the INS; and y could be estimated from the localizer signal and the altitude (the latter sensed by a radar altimeter). However, the usual situation is that not all of the states are sensed; furthermore, the sensed signals are often sufficiently noisy that some filtering is required. An electronic estimator could be constructed that estimates all of the states from the available measurements and these estimated states could then be fedback (see section on "Feedback of Estimated State").

Perkins[1] notes that the idea that a pilot is a "feedback controller" was not widely shared until World War II. In the last 25 years this idea has been extended to regard a skilled, experienced pilot as an "optimal feedback controller." In fact, many airline pilots regard the autoland systems developed for the wide-body transports with great respect (even awe), since they consistently make graceful landings even under difficult weather conditions. A good autopilot can serve as an instructor to a pilot flying a new aircraft. The autopilot designer may be said to learn how to fly a new aircraft by synthesizing feedback logic and then simulating the responses of the aircraft operating with this logic. If he is a good designer, the control histories for commanded outputs indicate good piloting technique. The simulated 747 S-turn above might qualify as an example.

Random Processes

Present-day control theory is also based on an understanding of random (or Stochastic) processes. This understanding began with Gauss about 150 years ago. His famous "Gaussian distribution" is the basis of much of our present day theory of random processes. Einstein also made a significant contribution in a paper on Brownian motion in 1904. Markov and many others followed Einstein in developing this theory.[21] In fact, the model of a random process used most frequently in control theory is called a "Gauss-Markov" process. Until 1959, this theory was based almost exclusively on "generalized harmonic analysis," created independently by G.I. Taylor and Norbert Wiener.[22,23]

Estimation

In 1960-61 Kalman and Bucy[24,25] showed how the Gauss-Markov model could be treated in the time-domain instead of the frequency-domain (as done by Taylor and Wiener), and gave a method of synthesizing linear estimators (or filters).

Their estimator is based on the concept of linear feedback of the estimate error. Suppose a set of measurements of outputs of the dynamic system (1) are made continuously. Let these measurements be the elements of a vector z, and the assume z is linearly related to the state vector of the system:

$$z = H \cdot x \tag{10}$$

$$\dot{z} = Az + By, \quad u = Cz,$$

y = (AIRSPEED, PITCH ANGLE, GLIDE-SLOPE DEVIATION)

u = (ELEVATOR, THROTTLE),

$$A = \begin{bmatrix} -3.74 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -12.64 & -4.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.870 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1.812 & -1.861 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -.220 \end{bmatrix}$$

$$B = \begin{bmatrix} -.670 & -.011 & 0 \\ .010 & .006 & 0 \\ -.002 & -.008 & -.009 \\ .021 & .077 & -.003 \\ -.269 & -.051 & -.011 \\ .339 & -.013 & .136 \\ .027 & .041 & .001 \end{bmatrix} \begin{matrix} \\ \leftarrow \text{NEGLIGIBLE} \\ \text{"} \\ \leftarrow \text{NEGLIGIBLE} \\ \\ \\ \leftarrow \text{NEGLIGIBLE} \end{matrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 & -.271 & 0 & 1 & 0 \\ .644 & -1.585 & -.905 & 1 & -.125 & .184 & 1 \end{bmatrix}$$

Fig. 12 Navion glide slope compensator of order 7 (35 parameters).

An "open-loop" estimator could be constructed based on the dynamic model of the system (1):

$$\dot{\hat{x}} = F \cdot \hat{x} + G \cdot u \tag{11}$$

where $\hat{x}$ = estimate of x.

This estimator would quickly develop errors because the model is never quite correct, and there will be random inputs to the real dynamic system (such as wind gusts on an airplane). However, it can be modified to track the measured quantities z as follows:

$$\dot{\hat{x}} = F \cdot \hat{x} + G \cdot u + K \cdot (z - H \cdot \hat{x}) \tag{12}$$

where K is a feedback gain matrix chosen to ensure that the estimate-error, $e = \hat{x} - x$, attenuates with time. Note that $z - H \cdot \hat{x}$ is the difference between the measurement vector z and the estimate of what z should be based on the current estimate of the state $\hat{x}$. Since u is a known signal it enters into the real system and the estimator in the same way. Assuming that F and G are known reasonably well, we may subtract Eq. (3) from Eq. (12), which implies the following description of the estimate-error:

$$\dot{e} = (F - KH) \cdot e \tag{13}$$

Thus the design problem is to pick K so that $F - KH$ represents a stable system, i.e., has eigenvalues with negative real parts. Kalman and Bucy showed how to pick K "optimally" by making a trade-off between the random errors in the measurements and the random inputs to the dynamic system. A "least-squares-fit" interpretation of the Kalman-Bucy filter is given in Ref. 26.

Battin developed the recursive-estimator independently, and his book[27] was the first to describe its use in space navigation. The more recent book, edited by Wertz, summarizes the state-of-the-art in spacecraft attitude determination and control,[28]

The Kalman-Bucy filter has become accepted in practice for navigation systems. However, its use for estimated-state-feedback (see below) is still not widely accepted because the resulting compensators are, in general, not robust to uncertainties or variations in the plant (see the section below on "Compensator Synthesis Using Parameter Optimization").

$$\dot{z} = Az + By, \quad u = Cz,$$

y = (AIRSPEED, PITCH ANGLE, GLIDE-SLOPE DEVIATION)

u = (ELEVATOR, THROTTLE),

$$A = \begin{bmatrix} -3.74 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2.80 & -2.59 \end{bmatrix}$$

$$B = \begin{bmatrix} -.589 & -.559 & -.180 \\ -.314 & .180 & -.082 \\ .412 & -.589 & .305 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 \\ .773 & .208 & .367 \end{bmatrix}$$

Fig. 13 Navion glide slope compensator of order 3 (20 parameters).

	RMS VALUES			
COMPENSATOR ORDER	u − uw FT/SEC	d FT	δe .01 RAD	δt FT/SEC²
7	2.02	3.45	.367	.274
4	2.03	3.46	.366	.275
3	2.09	3.54	.348	.268
2	2.18	3.71	.273	.250
1	2.30	5.27	.249	.293

Fig. 14 RMS outputs and controls for Navion glide slope compensators of orders 7, 4, 3, 2, and 1.

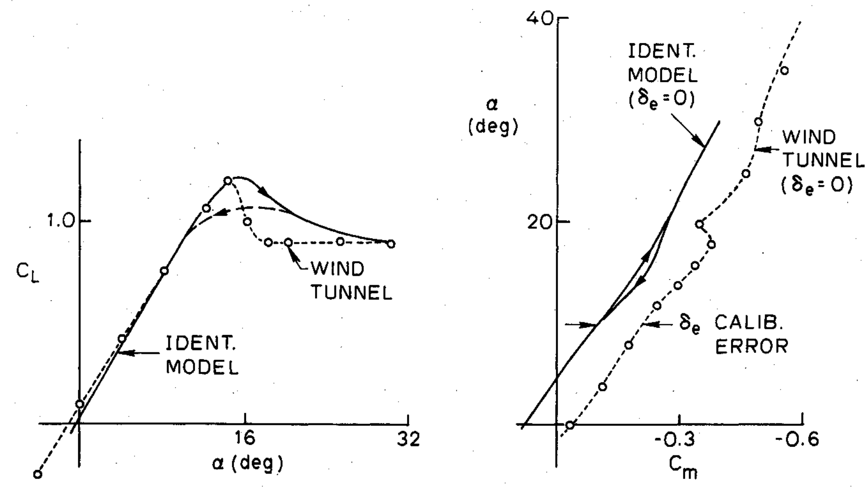


Fig. 15 Identified model vs wind tunnel data; lift and pitching moment coefficient vs angle of attack.[37]

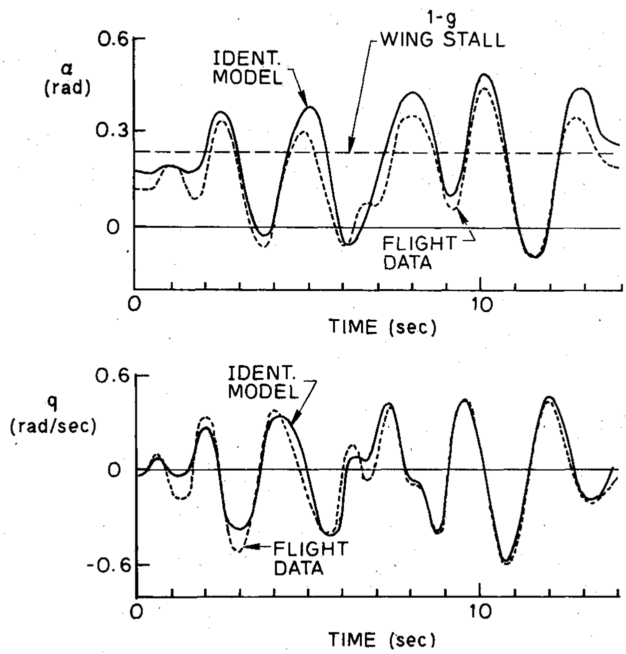


Fig. 16 Predictions of identified model vs flight data; angle of attack and pitch rate vs time.[37]

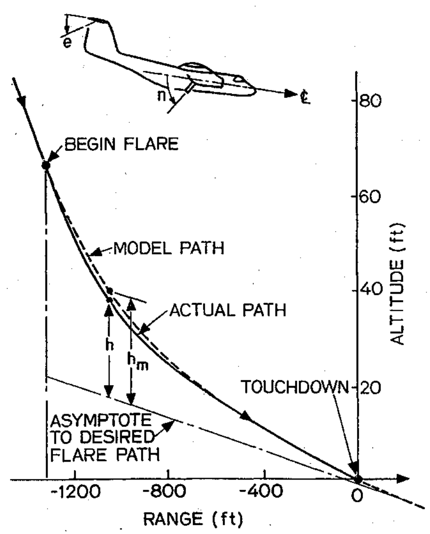


Fig. 17 Simulation of STOL aircraft landing flare using model-following logic.[38]

Feedback of Estimated State

State-feedback, as in Eq. (3), would require a measurement of each state. This would often be difficult and expensive; hence, it was natural to consider feeding back the estimated state, i.e.,

$$u = -C \cdot \hat{x} \qquad (14)$$

where the estimated state comes from an estimator of Eq. (12). This would appear to be a good control scheme if all of the states can be successfully estimated from the measurements z (i.e., the system is "observable" with the measurements z).

Equations (12) and (14) can be interpreted as a "dynamic compensator," where the measured vector z is the input and the control vector u is the output. Designing such a multi-input, multi-output compensator by "classical" methods involves an interative procedure called "successive loop closing." In the linear-quadratic synthesis procedure, all of the loops are closed simultaneously, and good closed-loop performance is obtained if the plant model is sufficiently accurate. One of the advantages of linear-quadratic synthesis for multi-input, multi-output systems is that the several controls are well-coordinated, and it is often rather difficult to obtain good coordination with the successive loop-closing procedure.

Equations (12) and (14) may be put into modal form for convenient implementation. Figure 12 shows such a modal form compensator for the Navion using three sensors (airspeed, pitch angle, and deviation from the glide slope) and using two controls (elevator and throttle).

One of the interesting predictions of this theory is that the eigenvalues of the controlled system will be the eigenvalues of the full-state-feedback regulator plus the eigenvalues of the estimate-error, i.e., the eigenvalues of

$$F - GC \text{ and } F - KH \qquad (15)$$

This is called the "separation principle" and was discovered independently in Refs. 28 and 29. This greatly simplifies the designer's task by separating the determination of the regulator gains C and the estimator gains K. The cited references also showed that estimated-state feedback using the "optimal" gains minimizes the expected-value of the performance index [Eq. (9)].

The material in this and the preceding sections on State Variable Representations, State Feedback and Optimal Control, Estimation, and Feedback of Estimated State are covered in detail in an excellent book by Kwakernaak and Sivan.[30]

Compensator Synthesis Using Parameter Optimization

One of the disadvantages of feeding back estimated state is that it requires an estimator (and hence a compensator) of the

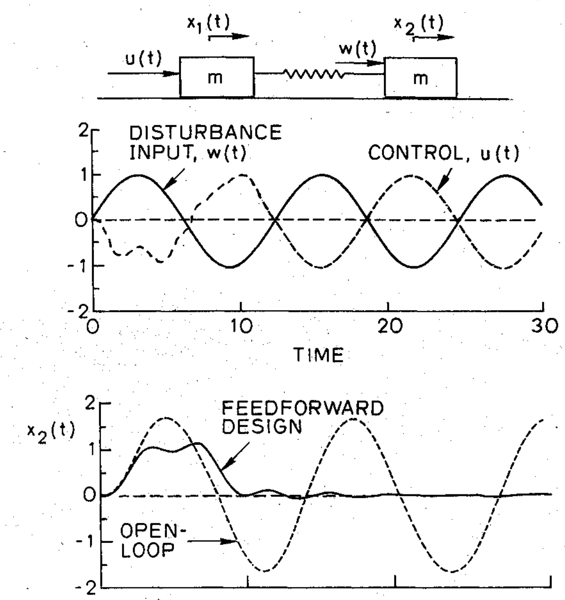


Fig. 18 Simulation of disturbance rejection.[36]

same order as the model of the plant. Experience with the classical design procedure demonstrated clearly that compensators of much lower order than the plant model are frequently quite satisfactory. In Refs. 31 through 34 methods were given for determining lower order compensators that minimize the expected value of the quadratic performance index [Eq. (9)], either with random initial conditions, or with steady-state random forcing functions. The order and the stucture of the compensator are chosen by the designer, as is the performance index. He then uses a numerical gradient scheme to find the compensator parameters that minimize the performance index. Care must be used in specifying the structure of the compensator, in that it must be a "minimum realization" form, i.e., a realization with the minimum number of parameters for that order and structure. The minimization procedure is much faster if analytical expressions can be found for the gradients; this was done in Refs. 34 and 35, where certain matrix Lyapunov equations are solved to find the gradient.

Figure 13 shows a third order compensator for the longitudinal motions of the Navion aircraft using the same three sensors as the seventh order compensator obtained by using estimated state feedback (see Fig. 12). Based on RMS output predictions, this much simpler compensator performances almost as well as the full order compensator (see Fig. 14).

Another disadvantage of the estimated-state compensator is that it relies heavily on the analytical plant model. If that model is inaccurate, the estimator is inaccurate and the "optimal" compensator can even produce unstable behavior of the closed-loop system! What is needed is a compensator that is "robust" to uncertainties in the plant model. Reference 35 has given a method for synthesizing robust compensators, using a weighted sum of quadratic performance indices. Each index is calculated using a different plant model but with the same control logic. Again, a numerical gradient scheme is used to find the optimal compensator parameters. Performance with the nominal plant parameters is sacrificed for stability over a range of values of the plant parameters.

Identification and Adaptive Control

It is difficult to synthesize a good control system if the plant model is inaccurate. Designing for parameter uncertainty is possible (see previous paragraph) but performance is lost when the parameters are nominal. Plant models are based on theory and experiment. One of the best experiments is to record inputs and outputs to the actual plant. If these records are reasonably accurate, the disturbance inputs to the plant are negligible compared to the control inputs, and the control inputs excite all the modes of the plant, these records can then be used to "identify" a plant model, using "least-square fit" techniques. There are many algorithms for doing this, and identification has become an important part of control synthesis.

Figure 15 compares a nonlinear model of an aircraft identified from flight data with wind tunnel data.[36] Note the hysteresis in the identified C_L vs α curve. Figure 16 compares predictions of responses using the identified model with flight data. Note the large angle-of-attack.

It is possible, in some cases to do identification in real time, and to modify the control law continuously based on this identification to improve controlled performance. Such a scheme is called an "adaptive control" scheme. The first successful adaptive controller for an airplane was used on the X-15. An adaptive autopilot is currently in use on the F-111 airplane. However, neither of these autopilots explicitly "identify" models of the airplane; instead they adjust gains using criteria related to phase and gain margins.

Model-Following, Tracking, and Disturbance Rejection

Model following logic may also be used for control synthesis. For example, we may wish to make a given aircraft respond to the pilot's controls like another aircraft. To do this, we connect the pilot's controls to an electronic model of the other aircraft. The controls are then moved by the autopilot to make the real outputs as close as possible to the model outputs. If the model can be changed easily, we call this a "variable-stability aircraft." Such aircraft are used for research and for training pilots to fly other aircraft that are more expensive to fly (like the space shuttle).

Model-following logic may also be used for "tracking" tasks like automatic flare control. The "model" in this case is a curve in space that starts (say) 50 ft above the runway tangent to the approach path, and touches down at a desired distance down the runway at a desired angle. Figure 17 shows a simulation of a STOL aircraft making an automatic landing tracking an exponential flare curve (from Ref. 37). Aircraft landings on a ship can be improved by estimating the ship's motions from sensors on the ship and a dynamic model of the ship.

The performance of radar trackers can be improved by using a dynamic model of the vehicle being tracked to estimate its "states." The control system can then predict the vehicle's motions a short time into the future, which makes for improved tracking. Even a very simple model like constant velocity can be helpful.

Disturbance rejection can sometimes be improved by using a dynamic model of the disturbance. For example, wind gusts are often modeled as first-order dynamic processes with white noise inputs. Another example is vibration isolation. If the frequency of the vibration source is known, an undamped oscillator "model" with that frequency can be used to estimate the "states" (magnitude and phase) of the disturbance, and this information can then be used to move controls to cancel the unwanted effect of the vibratory disturbance. Figure 18 shows an example of such a system, where a vibratory disturbance (say, an unbalanced rotor) is moving the right mass and we have a force control on the left mass.[35] In only one vibration period, the control logic estimates the magnitude and phase of the disturbance force and produces a control force that moves the left mass so that the spring force on the right mass is equal and opposite to the disturbance force.

Algorithms

As digital computers became more common, it became apparent that some people wrote more efficient and more reliable programs than others. In fact, it soon became ap-

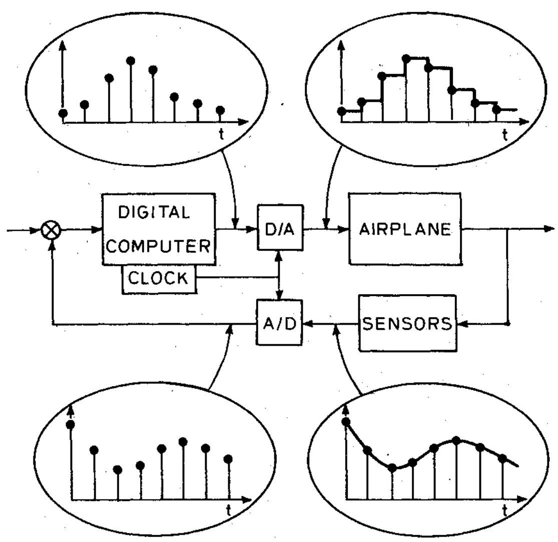


Fig. 19 Digital flight control concept.[46]

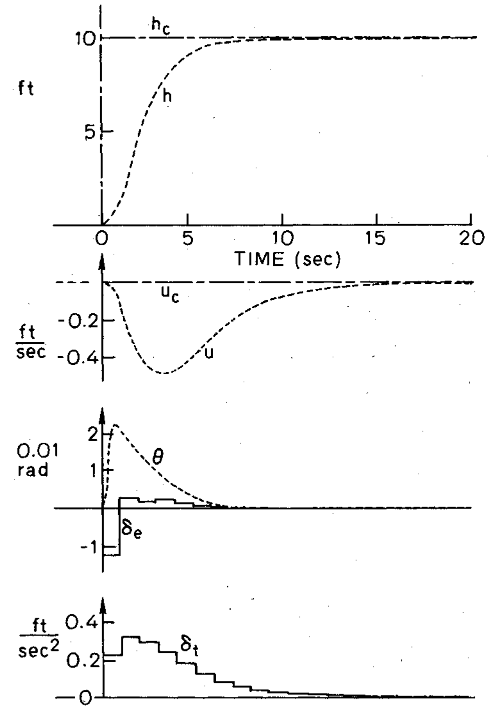


Fig. 20 Response of Navion to command for 10-ft increase in altitude using digital controller.

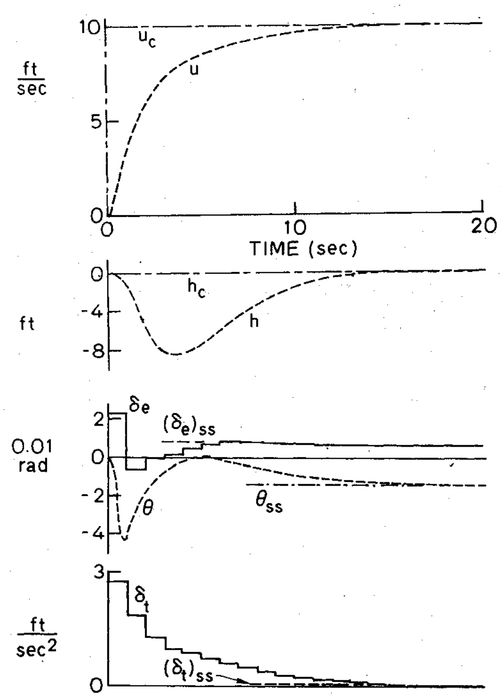


Fig. 21 Response of Navion to command for 10-ft/s increase in airspeed using digital controller.

parent that developing a good algorithm was an intellectually challenging proposition! A new group of applied mathematicians appeared who specialize in this area of algorithm development, and they have made substantial contributions to control theory.

One of the most significant algorithms for control theory is the MacFarlane-Potter algorithm for finding the optimal gain matrices for regulators and estimators.[38,39] It requires finding the eigenvalues and eigenvectors of large matrices. Fortunately, the remarkably efficient and reliable *QR* alrogithm for doing this was developed about the same time by Francis and Wilkerson.[40,41] Hall[42] brought these two algorithms together in a code he called "OPTSYS," which has been a mainstay of our research at Stanford since 1971.

Another group of algorithms has improved the reliability of time-varying Kalman filters; these are the so-called "square-root" or UDU algorithms.[43,44] They propagate the square-root of the symmetric covariance matrix instead of the covariance matrix, which givs nearly twice the accuracy for a given computer world-length as Kalman's original algorithm and prevents numerical truncation from producing a covariance matrix with negative eigenvalues.

Digital Control

A digital control system uses a digital computer to implement the control logic, and contains analog-to-digital and digital-to-analog converters (see Fig. 19). The first commercial digital computers in the 1950's were not fast enough or small enough to be carried on vehicles. The Apollo program spurred the development of smaller, faster computers for digital control of boosters and spacecraft in the 1960's. This technology was transferred to aircraft by NASA in the 1970's. The invention of micro-processors in the 1970's made digital computers very small, fast, and reliable. Many military and civilian aircraft now have digital control systems.

Franklin and Powell have written an excellent book on the special requirements and problems of digital control, which formerly was called "sampled-data" control.[45] For purposes of synthesizing digital controllers, the differential equation model of the dynamic system is transformed into a difference equation model where the controls are held constant between sampling times (a "zero-order-hold" model). Analysis is carried out in the complex z-plane instead of the complex s-plane, where $z = \exp(sT)$ and T is the sample period. Algorithms for synthesizing optimal digital controllers and estimators have been developed that are similar to the ones described above for analog (continuous) systems.[46]

Figure 20 and 21 show simulated responses of the Navion for commands in altitude and airspeed, using a digital controller with a one second sample period. Current digital controllers use much shorter sample periods (10 to 25 ms). The more sophisticated controllers use different sample periods in different loops, e.g., longer periods for outer loops and shorter periods for inner loops. Such multi-rate controllers are more readily designed using successive loop closures than LQ techniques.

Summary

New theoretical concepts, along with digital computers and inertial sensors, have produced big changes in navigation, guidance, and control in the last 25 years.

Optimal flight planning is becoming routine for long distance aircraft routes and for spacecraft and boosters.

Kalman-Bucy estimators have become accepted in practice for a great variety of navigation systems. They provide more accurate estimates of position and velocity and make possible the simultaneous use of many sensors.

Linear-quadratic (LQ) synthesis of autopilot logic is still not accepted in practice because of problems with robustness to uncertainties and variations in the vehicle model. However, the LQ regulator (with full state feedback) is widely admired for its ability to produce graceful flight paths with coordination of two or more controls. Current research is focused on improving the robustness of LQ autopilots.

Identification of vehicle models from flight tests is nearing acceptance in practice but still has a ways to go before it is a trusted tool. Adaptive control has been used on a few aircraft (very simple versions), but still needs further development before it is regarded as reliable.

Digital control has made possible more accurate and more sophisticated autopilot logic, and promises to be central in most improvements in navigation, guidance, and control in the future.

Acknowledgments

There have been thousands of important contributors to control theory over the past 25 years and my references contain fewer than a hundred names. My own knowledge of the field has been strongly shaped by my colleagues at Hughes (particularly Dick Edwards and Allen Puckett), and my colleagues and students at Harvard and Stanford (particularly Larry Ho, John Breakwell, Bob Cannon, Dan DeBra, Gene Franklin, Tom Kailath, and Dave Powell). I should also like to thank the editors of this journal for suggesting improvements in the manuscript.

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